COMPOUND ANGLES (Gr 12 only)

1: sin(A ± B) & cos(A ± B)

1 Complete:
(a) $\sin(A \pm B) =$
(b) $\cos(A \pm B) =$ (4)

2.1 Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$ to show that $\sin(A - B) = \sin A \cos B - \cos A \sin B$. (3)

2.2 Determine the value of $\sin(A + B)$ if $5\cos A - 4 = 0$ and $12 \tan B + 5 = 0$; A and $B \in [0°; 180°]$ (8)

3 Use the formula for $\cos(x - y)$ to show, without using a calculator, that: $\cos 15° = \frac{\sqrt{3} + 1}{2\sqrt{2}}$ (5)

4 Show that $\sin(A + B) - \sin(A - B) = 2\cos A \sin B$.
Hence calculate $\frac{\sin 105° - \sin 15°}{\sin 105° + \sin 15°}$ without using a calculator *(leave your answer in surd form)*. (6)

5.1 Prove that $\cos(A + B) - \cos(A - B) = -2\sin A \sin B$. (2)
Now factorise $\cos 5x - \cos x$. (4)

5.2 Evaluate
(a) $\cos 75° + \cos 15°$ without using a calculator. (5)
(b) $2\sin 195° . \sin 45°$ (6)

6 Simplify the following expression: $\frac{\sin 3x + \sin 7x}{\cos 3x + \cos 7x}$
[HINT: $3x = (5x - 2x)$ and $7x = (5x + 2x)$] (6)

7 If $\cos 61° = p$, express the following in terms of p:
(a) $\sin 209°$ (b) $\cos(-421°)$ (c) $\cos 1°$ (3)(3)(6)

8 Given that $\hat{P}$ and $\hat{Q}$ are both acute, solve for P and Q if
$\sin P \sin Q - \cos P \cos Q = \frac{1}{2}$
and $\sin P \cos Q - \cos P \sin Q = \frac{1}{2}$ (6)

9 Simplify:
(a) $\sin 70° \cos 40° - \cos 70° \sin 40°$ (2)
(b) $\sin(90° - x) \cos x + \cos(90° - x) \sin x$ (2)
(c) $\cos(x + 70°) \cos(x + 40°) + \sin(x + 70°) \sin(x + 40°)$ (3)
(d) $\cos 70° \cos 10° + \cos 20° \cos 80°$ (4)

2: sin 2A & cos 2A

1 (a) Derive the formula for $\sin 2A$ from the formula for $\sin(A + B)$. (2)
(b) Derive three formulae for $\cos 2A$ from the formula for $\cos(A + B)$. (4)
(c) Now, deduce:
(1) $\sin 4A =$ *. . . in terms of 2A* (2)
(2) $\sin A =$ *. . . in terms of* $\frac{A}{2}$ (2)
(3) $\cos 6A =$ *. . . in terms of 3A* (2)

2 If $90° < A < 360°$ and $\tan A = \frac{2}{3}$, determine without using a calculator *(leave your answer in surd form where applicable)*:
(a) $\sin A$ (b) $\cos 2A - \sin 2A$ (3)(4)

3.1 If $\sin 9° = a$, calculate without using a calculator, the value of $\sin 18°$ in terms of a. (5)

3.2 Given: $\cos x = t$
Express each of the following in terms of t:
(a) $\cos(180° + x)$ (b) $\sin^2 x$ (c) $\cos 2x$ (2)(1)(2)

3.3 If $\cos 18° = k$, express the following in terms of k:
(a) $\sin 18°$ (b) $\cos 162°$ (2)(2)
(c) $\tan 108°$ (d) $\cos(-36°)$ (2)(2)

4 Calculate, without using a calculator, the value of $\sin(90° + 2\theta)$ if $3 \cos\theta = 1$ (6)

5.1 Prove the following identity:
$\sin(45° + x) . \sin(45° - x) = \frac{1}{2} \cos 2x$ (5)

5.2 Hence determine the maximum value of $\sin(45° + x) . \sin(45° - x)$ and the corresponding value(s) of $x \in [0°; 180°]$. (4)

3: Prove the following identities :

1 $\frac{\sin 2x}{\sin x} . \frac{\cos 2x}{\cos x} . \frac{\tan 2x}{\tan x} = 4\cos^2 x$ (5)

2 $\frac{1 - \cos 2\theta}{\sin 2\theta} = \tan\theta$

2.1 Hence deduce that $\tan 15° = 2 - \sqrt{3}$ (8)

3 $\frac{\sin 2\theta - \cos\theta}{\sin\theta - \cos 2\theta} = \frac{\cos\theta}{1 + \sin\theta}$ (5)

4 $\frac{\sin x - \sin 2x}{\cos x - \cos 2x - 1} = \tan x$ (5)

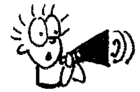
5 $\frac{1 - \cos 2x - \sin x}{\sin 2x - \cos x} = \tan x$ (7)

6 $\frac{\sin y + \sin 2y}{\cos y + \cos 2y + 1} = \tan y$ (5)

7 $\frac{\cos 2x - \cos x}{\sin 2x + \sin x} = \frac{\cos x - 1}{\sin x}$ (4)

8 $\frac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$ (5)

9 $\frac{\cos x - \cos 2x + 2}{3\sin x - \sin 2x} = \frac{1 + \cos x}{\sin x}$ (7)



10 $\frac{\sin 2\theta . \tan\theta}{\cos 2\theta + 1} = \tan^2\theta$ (4)

11 $\frac{1 - \sin 2x}{\sin x - \cos x} = \sin x - \cos x$ (2)
11.1 For which values of x is this identity undefined ? (?)

12 $\sin\theta \cos\theta \cos 2\theta \cos 4\theta = \frac{1}{8} \sin 8\theta$ (5)

13 Prove the identity : $\cos 3A = 4\cos^3 A - 3\cos A$ (5)

14 Prove that : $\sin 3A = 3\sin A - 4\sin^3 A$. (5)
14.1 Hence, determine the general solution for A if:
$8\sin^3 A = 6\sin A + 1$. (7)

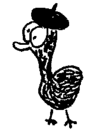
4: Simplify the following expressions:

(without the use of a calculator)

1. $(\sin 15° + \cos 15°)^2$ **2.** $\frac{\sin 40° \cos 40°}{\cos 10°}$ (4)(3)

3 $\frac{\tan(-330°) . \sin 120° . \sin 250°}{\cos 215° . \sin 325°}$ (6)

4 $\left(\sin\frac{A}{2} - \cos\frac{A}{2}\right)\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)$



5 $\frac{(\sin 75° - \cos 75°)(\sin 75° + \cos 75°)}{\tan(-150°) \sin 300°}$ (8)

6 $\frac{\sin 15° \cos 15°}{\cos(45° - x)\cos x - \sin(45° - x)\sin x}$

7 Evaluate $\frac{\sin 6x}{\sin 2x} - \frac{\cos 6x}{\cos 2x}$ (4)
7.1 For which values of $x \in [0°; 180°]$, will the expression above be undefined? (5)

5: Solve the following equations . . .

1 $\sin 2x = \tan 225° \cos 210°$ for $x \in [0°; 180°]$ (7)
2 $\sin 2x + \cos x = 0$ for $x \in [0°; 180°]$ (5)
3 $\sin 2x + \sin^2 x = 0$ for $x \in [0°; 180°]$ (7)
4 $\cos 2x - 5\cos x - 2 = 0$ *(general solution)* (6)
5 $\cos 2x + 7\sin x - 4 = 0$ for $x \in [0°; 180°]$ (8)
6 $\cos 2\theta + 3\sin^2\theta - 4\sin\theta + 2 = 0$ for $\theta \in [-180°; 180°]$ (7)
7 $\cos 2x - 7\cos x . \tan x = 4$ and $x \in [-180°; 90°]$ (8)
8 $\sin 2\theta = \cos(-3\theta)$ *(general solution)* (6)
9 $\sin(x + 30°) = -\cos 2x$ *(general solution)* (8)
10 $\sin 2x - \sin x + 2\sin^2 x - \cos x = 0$ *(general solution)* (8)
11 $\sin 2x + \sin x = 6\cos x + 3$ for $-180° \le x \le 0°$ (7)
12 $\frac{\sin 2x}{\cos 2x - 1} = 1$ $x \in [90°; 270°]$ (7)
13 $4\sin x \cos x = 1$ $x \in [0°; 90°]$ (4)
14 $6\sin^2 x + 2\sin 2x = 1$ and $-90° \le x \le 90°$ (10)

COMPOUND ANGLES

1: $\sin(A \pm B)$ & $\cos(A \pm B)$

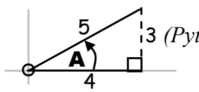
1 (a) $\mathbf{\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B}$ ◄
(b) $\mathbf{\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B}$ ◄

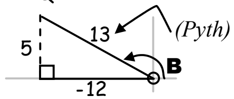
2.1 $\sin(A + B) = \cos[90° - (A + B)]$
$= \cos(90° - A - B)$
$= \cos[(90° - A) - B]$ . . . the cos of the **difference** of 2 angles !
$= \cos(90° - A)\cos B + \sin(90° - A)\sin B$
$= \mathbf{\sin A \cos B + \cos A \sin B}$ ◄

2.2 $\cos A = \frac{4}{5}$ & $A \in [0°; 180°]$

& $\tan B = -\frac{5}{12}$ & $B \in [0°; 180°]$

$\therefore$ $\hat{A}$ in 1st Quad.: $\hat{B}$ in 2nd Quad.:





$\sin(A + B) = \sin A \cos B + \cos A \sin B$
$= \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)$
$= -\frac{36}{65} + \frac{4}{13}$
$= \frac{-36 + 20}{65}$
$= \frac{-16}{65}$ ◄

3 $\cos(x - y) = \cos x \cos y + \sin x \sin y$
$\therefore \cos 15° = \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$
$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$
$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$ ◄

4 $\sin(A + B) = \sin A \cos B + \cos A \sin B$ . . . ❶
& $\sin(A - B) = \sin A \cos B - \cos A \sin B$ . . . ❷
$\therefore \sin(A + B) - \sin(A - B) = \cos A \sin B - (-\cos A \sin B)$ . . . ❶ - ❷
$= 2\cos A \sin B$ ◄

Similarly : $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$

Let $105° = 60° + 45°$ *(i.e. A = 60° & B = 45°)*
& $15° = 60° - 45°$

then $\frac{\sin 105° - \sin 15°}{\sin 105° + \sin 15°} = \frac{2\cos 60° \sin 45°}{2\sin 60° \cos 45°} = \frac{2\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}{2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{3}}$ ◄

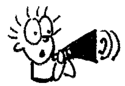
5.1 $\cos(A + B) = \cos A \cos B - \sin A \sin B$ . . . ❶
& $\cos(A - B) = \cos A \cos B + \sin A \sin B$ . . . ❷
$\therefore \cos(A + B) - \cos(A - B) = -\sin A \sin B - \sin A \sin B$ . . . ❶ - ❷
$= -2\sin A \sin B$ ◄

Let $5x = 3x + 2$; then $x = 3x - 2x$ *(i.e. A = 3x & B = 2x)*

$\therefore \cos 5x - \cos x = \cos(3x + 2x) - \cos(3x - 2x)$
$= -2\sin 3x \sin 2x$ ◄

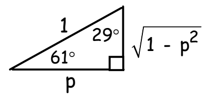
5.2 (a) Similarly to above formula:
$\cos(A + B) + \cos(A - B) = 2\cos A \cos B$
$\therefore \cos 75° + \cos 15° = \cos(45° + 30°) + \cos(45° - 30°)$
$= 2\cos 45° \cos 30°$ . . . *A = 45° & B = 30°*
$= 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$
$= \sqrt{\frac{3}{2}}$ ◄

(b) 5.1: $-2\sin A \sin B = \cos(A + B) - \cos(A - B)$
➡ $2\sin A \sin B = \cos(A - B) - \cos(A + B)$
$\therefore 2\sin 195°.\sin 45° = \cos(195° - 45°) - \cos(195° + 45°)$
$= \cos 150° - \cos 240°$
$= -\cos 30° - (-\cos 60°)$
$= -\frac{\sqrt{3}}{2} + \frac{1}{2}$
$= \frac{-\sqrt{3} + 1}{2}$ ◄



6 $\frac{\sin(5x - 2x) + \sin(5x + 2x)}{\cos(5x - 2x) + \cos(5x + 2x)}$
$= \frac{2\sin 5x \cos 2x}{2\cos 5x \cos 2x}$ . . . see formula in Q4
$= \frac{\sin 5x}{\cos 5x}$
$= \tan 5x$ ◄

7 $\cos 61° = \frac{p}{1}$ . . .



$\sqrt{1 - p^2}$ . . . *(Pythag)*

(a) $\sin 209° = -\sin 29° = -p$ ◄
(b) $\cos(-421°) = \cos(-61°) = \cos 61° = p$ ◄
(c) $\cos 1° = \cos(61° - 60°)$!!!
$= \cos 61° \cos 60° + \sin 61° \sin 60°$
$= p.\frac{1}{2} + \sqrt{1 - p^2}.\frac{\sqrt{3}}{2}$
$= \frac{p + \sqrt{1 - p^2}.\sqrt{3}}{2}$ ◄

8 ❶ : $-\cos(P + Q) = \frac{1}{2}$ ❷ : $\sin(P - Q) = \frac{1}{2}$
$\therefore \cos(P + Q) = -\frac{1}{2}$ $\therefore P - Q = 30°$ . . . ❷
$\therefore P + Q = 180° - 60°$
$= 120°$ . . . ❶
❶ + ❷ : $\therefore 2P = 150°$
$\therefore P = 75°$ & $Q = 45°$

Applying formulae for cos(P + Q) & sin(P - Q)

9 (a) $\sin(70° - 40°)$ . . . *formula for sin(A - B)*
$= \sin 30°$
$= \frac{1}{2}$ ◄
(b) $\sin[(90° - x) + x]$ . . . *formula for sin(A + B)*
$= \sin 90°$
$= 1$ ◄
(c) $\cos[(x + 70°) - (x + 40°)]$ . . . *formula for cos(A - B)*
$= \cos 30°$
$= \frac{\sqrt{3}}{2}$ ◄
(d) $\cos 70° \cos 10° + \sin 70° \sin 10°$ . . . *formula for cos(A - B)*
$= \cos[70° - 10°]$
$= \cos 60°$
$= \frac{1}{2}$ ◄

2: $\sin 2A$ & $\cos 2A$

xy + yx = 2xy !

1 (a) $\mathbf{\sin 2A} = \sin(A + A) = \sin A \cos A + \cos A \sin A$
$= \mathbf{2\sin A \cos A}$ ◄

(b) $\mathbf{\cos 2A} = \cos(A + A) = \cos A \cos A - \sin A \sin A$
$= \mathbf{\cos^2 A - \sin^2 A}$ ◄

But also, $\sin^2 A + \cos^2 A = 1$ ➡ $\cos^2 A = 1 - \sin^2 A$
and $\sin^2 A = 1 - \cos^2 A$

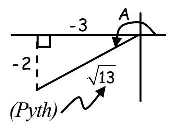
$\therefore \mathbf{\cos 2A} = (1 - \sin^2 A) - \sin^2 A = \mathbf{1 - 2\sin^2 A}$ ◄

or $\mathbf{\cos 2A} = \cos^2 A - (1 - \cos^2 A)$
$= \cos^2 A - 1 + \cos^2 A$
$= \mathbf{2\cos^2 A - 1}$ ◄

(c) (1) $\sin 4A = 2\sin 2A \cos 2A$
(2) $\sin A = \sin 2\left(\frac{A}{2}\right) = 2\sin\frac{A}{2}\cos\frac{A}{2}$
(3) $\cos 6A = \cos 2(3A) = \cos^2 3A - \sin^2 3A$

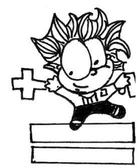
2 $90° < A < 360°$; $\tan A = +\frac{2}{3}$

$\therefore$ $\hat{A}$ in 3rd Quadrant :

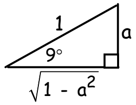


(a) $\sin A = -\frac{2}{\sqrt{13}}$

(b) $\cos 2A - \sin 2A$
$= 2\cos^2 A - 1 - 2\sin A \cos A$
$= 2\left(-\frac{3}{\sqrt{13}}\right)^2 - 1 - 2\left(-\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)$
$= 2\left(\frac{9}{13}\right) - 1 - \frac{12}{13}$
$= \frac{18 - 13 - 12}{13}$
$= -\frac{7}{13}$ ◄

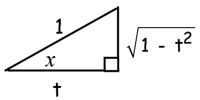


3.1 $\sin 9° = \frac{a}{1}$ . . .



$\therefore \sin 18° = \sin 2(9°) = 2\sin 9° \cos 9° = 2a.\sqrt{1 - a^2}$ ◄

3.2 $\cos x = \frac{t}{1}$

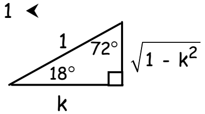


(a) $\cos(180° + x) = -\cos x = -t$ ◄

(b) $\sin^2 x = 1 - \cos^2 x = 1 - t^2$ ◄

(c) $\cos 2x = 2\cos^2 x - 1 = 2t^2 - 1$ ◄

3.3 $\cos 18° = \frac{k}{1}$



(a) $\sin 18° = \sqrt{1 - k^2}$ ◄

(b) $\cos 162° = -\cos 18° = -k$ ◄

(c) $\tan 108° = -\tan 72° = -\frac{k}{\sqrt{1 - k^2}}$ ◄

(d) $\cos(-36°) = \cos 36° = \cos 2(18°)$
$= 2\cos^2 18° - 1$
$= 2k^2 - 1$ ◄

4 $\sin(90° + 2\theta) = \cos 2\theta$ & $\cos\theta = \frac{1}{3}$

$= 2\cos^2\theta - 1$

$= 2\left(\frac{1}{3}\right)^2 - 1$

$= \frac{2}{9} - 1$

$= -\frac{7}{9}$

5.1 **LHS** $= (\sin 45° \cos x + \cos 45° \sin x)(\sin 45° \cos x - \cos 45° \sin x)$

$= \sin^2 45° \cos^2 x - \cos^2 45° \sin^2 x$ . . . $(a + b)(a - b) = a^2 - b^2$

$= \left(\frac{1}{\sqrt{2}}\right)^2 \cos^2 x - \left(\frac{1}{\sqrt{2}}\right)^2 \sin^2 x$

$= \frac{1}{2}\cos^2 x - \frac{1}{2}\sin^2 x$

$= \frac{1}{2}(\cos^2 x - \sin^2 x)$

$= \frac{1}{2}\cos 2x =$ **RHS** ◄

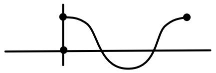
5.2 $\therefore$ Maximum value of $\sin(45° + x) . \sin(45° - x)$

= Maximum value of $\frac{1}{2}\cos 2x$

$= \frac{1}{2}(\mathbf{1})$ . . . *maximum value of* $\cos\theta = 1$!

$= \frac{1}{2}$ ◄

when $\cos 2x = 1$



$\therefore 2x = 0°$ or $360°$ . . . $2x \in [0°; 360°]$

$\therefore x = 0°$ or $180°$ . . . $x \in [0°; 180°]$

3: Proving identities :

1 **LHS** $= \frac{2\cancel{\sin x} . \cancel{\cos x}}{\cancel{\sin x}} . \frac{\cancel{\cos 2x}}{\cancel{\cos x}} . \frac{\sin 2x}{\cancel{\cos 2x} . \frac{\sin x}{\cos x}}$

$= \frac{2.2\cancel{\sin x}\cos x}{1} \times \frac{\cos x}{\cancel{\sin x}}$

$= 4\cos^2 x =$ **RHS** ◄

2 **LHS** $= \frac{1 - \mathbf{(1 - 2\sin^2\theta)}}{2\sin\theta\cos\theta} = \frac{2\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta =$ **RHS** ◄

2.1 $\tan 15° = \frac{1 - \cos 30°}{\sin 30°}$

$= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} \left(\times \frac{\mathbf{2}}{\mathbf{2}}\right)$

$= \frac{2 - \sqrt{3}}{1}$

$= 2 - \sqrt{3}$ ◄

3 **LHS** $= \frac{2\sin\theta\cos\theta - \cos\theta}{\sin\theta - \mathbf{(1 - 2\sin^2\theta)}}$

$= \frac{\cos\theta(2\sin\theta - 1)}{\sin\theta - 1 + 2\sin^2\theta}$

$= \frac{\cos\theta(2\sin\theta - 1)}{2\sin^2\theta + \sin\theta - 1}$

$= \frac{\cos\theta(2\sin\theta - 1)}{(2\sin\theta - 1)(\sin\theta + 1)}$

$= \frac{\cos\theta}{1 + \sin\theta} =$ **RHS** ◄

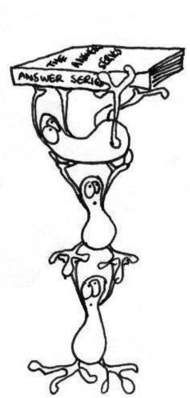
4 **LHS** $= \frac{\sin x - 2\sin x\cos x}{\cos x - \mathbf{(2\cos^2 x - 1)} - 1}$

$= \frac{\sin x(1 - 2\cos x)}{-2\cos^2 x + \cos x}$

$= \frac{-\sin x(2\cos x - 1)}{-\cos x(2\cos x - 1)}$

$= \frac{\sin x}{\cos x}$

$= \tan x =$ **RHS** ◄



5 **LHS** $= \frac{1 - \mathbf{(1 - 2\sin^2 x)} - \sin x}{2\sin x\cos x - \cos x}$

$= \frac{2\sin^2 x - \sin x}{\cos x(2\sin x - 1)}$

$= \frac{\sin x(2\sin x - 1)}{\cos x(2\sin x - 1)}$

$= \frac{\sin x}{\cos x}$

$= \tan x =$ **RHS** ◄

6 **LHS** $= \frac{\sin y + 2\sin y\cos y}{\cos y + \mathbf{2\cos^2 y - 1} + 1}$

$= \frac{\sin y(1 + 2\cos y)}{\cos y(1 + 2\cos y)}$

$= \tan y =$ **RHS** ◄

7 **LHS** $= \frac{\mathbf{2\cos^2 x - 1} - \cos x}{2\sin x\cos x + \sin x}$

$= \frac{2\cos^2 x - \cos x - 1}{\sin x(2\cos x + 1)}$

$= \frac{(2\cos x + 1)(\cos x - 1)}{\sin x(2\cos x + 1)}$

$= \frac{\cos x - 1}{\sin x} =$ **RHS** ◄

8 **LHS** $= \frac{2.\frac{\sin x}{\cos x} - 2\sin x\cos x}{2\sin^2 x}\left(\times \frac{\mathbf{\cos x}}{\mathbf{\cos x}}\right)$

OR: $\frac{2\sin x - 2\sin x\cos^2 x}{\cos x} \times \frac{1}{2\sin^2 x}$

$= \frac{2\sin x - 2\sin x\cos^2 x}{2\sin^2 x . \cos x}$

$= \frac{2\sin x(1 - \cos^2 x)}{2\sin^2 x . \cos x}$

$= \frac{\cancel{2}\sin x . \cancel{\sin^2} x}{\cancel{2}\cancel{\sin^2} x . \cos x}$

$= \frac{\sin x}{\cos x}$

$= \tan x =$ **RHS** ◄

9 **LHS** $= \frac{\cos x - \mathbf{(2\cos^2 x - 1)} + 2}{3\sin x - 2\sin x\cos x}$

$= \frac{\cos x - 2\cos^2 x + 3}{\sin x(3 - 2\cos x)}$

$= \frac{3 + \cos x - 2\cos^2 x}{\sin x(3 - 2\cos x)}$

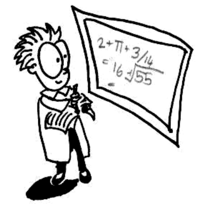
$= \frac{(3 - 2\cos x)(1 + \cos x)}{\sin x(3 - 2\cos x)}$

$= \frac{1 + \cos x}{\sin x} =$ **RHS** ◄

10 **LHS** $= \frac{2\sin\theta\cos\theta . \frac{\sin\theta}{\cos\theta}}{\mathbf{2\cos^2\theta - 1} + 1}$

$= \frac{2\sin^2\theta}{2\cos^2\theta}$

$= \tan^2\theta =$ **RHS** ◄



11 LHS $= \dfrac{\sin^2 x + \cos^2 x - 2\sin x\cos x}{\sin x - \cos x}$

$= \dfrac{\sin^2 x - 2\sin x\cos x + \cos^2 x}{\sin x - \cos x}$

$= \dfrac{(\sin x - \cos x)^2}{\sin x - \cos x}$

$= \sin x - \cos x =$ **RHS** ◄

11.1 Undefined when $\sin x - \cos x = 0$

$\therefore\ \sin x = \cos x$

$(\div \cos x)$ $\therefore\ \tan x = 1$

$\therefore\ x = 45° + n(180°),\ n \in \mathbb{Z}$ ◄

12 **RHS** $= \frac{1}{8}$ **sin 2(4θ)**

$= \frac{1}{8}.\mathbf{2\sin 4\theta \cos 4\theta}$

$= \frac{1}{8}.2.\mathbf{\sin 2(2\theta)}.\cos 4\theta$

$= \frac{1}{8}.2.\mathbf{2\sin 2\theta . \cos 2\theta}.\cos 4\theta$

$= \frac{1}{8}.2.2.\mathbf{2\sin\theta . \cos\theta}.\cos 2\theta.\cos 4\theta$

$= \sin\theta.\cos\theta.\cos 2\theta.\cos 4\theta$

$=$ **LHS** ◄

13 **LHS** $= \cos 3A$

$= \cos(2A + A)$

$= \cos 2A\cos A - \sin 2A\sin A$

$= (2\cos^2 A - 1)\cos A - 2\sin^2 A\cos A$ . . .

(sin 2A = 2sin A cos A)

$= 2\cos^3 A - \cos A - 2\cos A(1 - \cos^2 A)$

$= 2\cos^3 A - \cos A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A =$ **RHS** ◄

14 **LHS** $= \sin 3A = \sin(2A + A)$

$= \sin 2A\cos A + \cos 2A\sin A$

$= 2\sin A\cos^2 A + (1 - 2\sin^2 A).\sin A$

$= 2\sin A(1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A - 2\sin^3 A + \sin A - 2\sin^3 A$

$= 3\sin A - 4\sin^3 A =$ **RHS** ◄

14.1 $8\sin^3 A - 6\sin A = 1$

$\div(-2)$ $-4\sin^3 A + 3\sin A = -\frac{1}{2}$

$\therefore\ \sin 3A = -\frac{1}{2}$ . . . see no. 14

$\therefore\ 3A = 180° + 30° + n(360°),\ n \in \mathbb{Z}$ **. . . III**

$= 210° + n(360°)$

$\therefore\ A = 70° + n(120°)$ ◄

or $3A = 360° - 30° + n(360°),\ n \in \mathbb{Z}$ **. . . IV**

$= 330° + n(360°)$

$\therefore\ A = 110° + n(120°)$ ◄

4: Simplifying expressions :

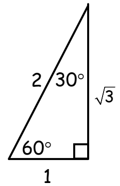
1 $\sin^2 15° + 2\sin 15°\cos 15° + \cos^2 15°$

$= \mathbf{\sin^2 15° + \cos^2 15°} + \sin 2(15°)$

. . . $\sin^2\theta + \cos^2\theta = 1$

$= 1 + \sin 30°$

$= 1\frac{1}{2}$ ◄ . . . $\sin 30° = \frac{1}{2}$



2 $\dfrac{\frac{1}{2}.2\sin 40°\cos 40°}{\sin 80°}$

$= \dfrac{\frac{1}{2}.\sin 80°}{\sin 80°}$

$= \frac{1}{2}$ ◄

OR: The denominator
$= \sin 80°$
$= \sin 2(40°)$
$= 2\sin 40°\cos 40°$
$\therefore$ Expr. $= \dfrac{\sin 40°\cos 40°}{2\sin 40°\cos 40°} = \frac{1}{2}$ ◄

3 Exp. $= \dfrac{\tan 30° . \sin 60° . (-\sin 70°)}{(-\cos 35°).(-\sin 35°)}$

$= -\dfrac{\frac{1}{\sqrt{3}}.\frac{\sqrt{3}}{2}.\sin 2(35°)}{+\cos 35°.\sin 35°}$

$= -\dfrac{\frac{1}{2}.2\sin 35°\cos 35°}{\cos 35°.\sin 35°}$

$= -1$ ◄

4 $\sin^2\frac{A}{2} - \cos^2\frac{A}{2} = -\left(\cos^2\frac{A}{2} - \sin^2\frac{A}{2}\right)$

$= -\cos A$ ◄ . . . *cos 2A formula*

5 Expression $= \dfrac{\sin^2 75° - \cos^2 75°}{\tan 30°.(-\sin 60°)}$

$= \dfrac{-\cos 2(75°)}{\frac{1}{\sqrt{3}}.\left(-\frac{\sqrt{3}}{2}\right)}$ . . . $\left(\begin{aligned}&\sin^2\theta - \cos^2\theta\\ &= -(\cos^2\theta - \sin^2\theta)\\ &= -\cos 2\theta\end{aligned}\right)$

$= \dfrac{-\cos 150°}{-\frac{1}{2}}$ . . . $\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$= \dfrac{-\left(-\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}}$ $\left(\times\frac{2}{2}\right)$

$= -\sqrt{3}$ ◄

6 $\dfrac{\frac{1}{2}.2\sin 15°\cos 15°}{\cos[(45° - x) + x]} = \dfrac{\frac{1}{2}.\sin 30°}{\cos 45°} = \dfrac{\frac{1}{2}.\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \dfrac{\sqrt{2}}{4}$ ◄

7 Exp. $= \dfrac{\sin 6x.\cos 2x - \cos 6x.\sin 2x}{\sin 2x.\cos 2x}$

$= \dfrac{\sin(6x - 2x)}{\sin 2x.\cos 2x}$ ·······► $= \sin 4x$
$= \sin 2(2x)$
$= 2\sin 2x\cos 2x$

$= \dfrac{2\sin 2x.\cos 2x}{\sin 2x.\cos 2x}$

$= 2$ ◄

7.1 **The expression is undefined when**

$\sin 2x = 0$ **OR** $\cos 2x = 0$ . . .

. . . **i.e. when the denominator = 0**

$\therefore\ 2x = 0° + n(180°)$ $\therefore\ 2x = 90° + n(180°)$

$\therefore\ x = 0° + n(90°)$ $\therefore\ x = 45° + n(90°)$

$\therefore$ When $x = 0°;\ 45°;\ 90°;\ 135°$ or $180°$ ◄

5: Solving equations . . .

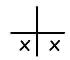
1 $\sin 2x = (+\tan 45°)(-\cos 30°)$

$= (+1)\left(-\frac{\sqrt{3}}{2}\right)$

$= -\frac{\sqrt{3}}{2}$ . . .

Note: $x \in [0°; 180°]$ ➡ $2x \in [0°; 360°]$

$\therefore\ 2x = 240°$ or $300°$



; ref L = 60°

$\therefore\ x = 120°$ or $150°$ ◄

2 $\mathbf{2\sin x\cos x} + \cos x = 0$

$\therefore\ \cos x(2\sin x + 1) = 0$

$\therefore\ \cos x = 0$ or $\sin x = -\frac{1}{2}$ for $x \in [0°; 180°]$

$\therefore\ x = 90°$ ◄ . . . $\sin x > 0$ for $x \in [0°; 180°]$

3 $\mathbf{2\sin x\cos x} + \sin^2 x = 0$

$\therefore\ \sin x(2\cos x + \sin x) = 0$

$\therefore\ \sin x = 0$ **OR** $2\cos x = -\sin x$

$\therefore\ x = 0°$ or $180°$ ◄ $\div(-\cos x)$ $\therefore\ -2 = \tan x$

$\therefore\ x = 180° - 63{,}43°$

$= 116{,}57°$ ◄

4 $\mathbf{2\cos^2 x - 1} - 5\cos x - 2 = 0$

$\therefore\ 2\cos^2 x - 5\cos x - 3 = 0$

$\therefore\ (2\cos x + 1)(\cos x - 3) = 0$

$\therefore\ \cos x = -\frac{1}{2}$ . . . $\cos x \neq 3$ $\because -1 \leq \cos x \leq 1$ *for all θ!*

$\therefore\ x = \pm 120° + n(360°),\ n \in \mathbb{Z}$ ◄

5 $\mathbf{1 - 2\sin^2 x} + 7\sin x - 4 = 0$

$\therefore\ 2\sin^2 x - 7\sin x + 3 = 0$

$\therefore\ (2\sin x - 1)(\sin x - 3) = 0$

$\therefore\ \sin x = \frac{1}{2}$ . . . $\sin x \neq 3$ $\because -1 \leq \sin x \leq 1$ *for all θ!*

$\therefore\ x = 30°$ or $150°$ ◄

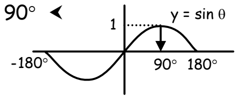
6 $\mathbf{1 - 2sin^2}\theta + 3\sin^2\theta - 4\sin\theta + 2 = 0$

$\therefore \sin^2\theta - 4\sin\theta + 3 = 0$

$\therefore (\sin\theta - 1)(\sin\theta - 3) = 0$

$\therefore \sin\theta = 1$ . . . $\sin\theta \neq 3$!

$\therefore \theta = 90°$ ◂



7 $\mathbf{1 - 2sin^2x} - 7\cos x \cdot \frac{\sin x}{\cos x} - 4 = 0$

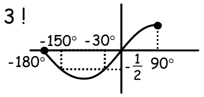
$\therefore -2\sin^2 x - 7\sin x - 3 = 0$

$\therefore 2\sin^2 x + 7\sin x + 3 = 0$

$\therefore (2\sin x + 1)(\sin x + 3) = 0$

$\therefore \sin x = -\frac{1}{2}$ . . . $\sin x \neq -3$!

$\therefore x = -150°$ or $-30°$ ◂



8 $\sin 2\theta = +\cos 3\theta$ . . . *cos (- θ) = cos θ*

$= +\sin(90° - 3\theta)$ *the reference angle*

$\therefore 2\theta = 90° - 3\theta + n(360°),\ n \in \mathbb{Z}$ . . . **I**

$\therefore 5\theta = 90° + n(360°)$

$\therefore \theta = 18° + n(72°)$ ◂

OR : $2\theta = 180° - (90° - 3\theta) + n(360°),\ n \in \mathbb{Z}$ . . . **II**

$= 90° + 3\theta + n(360°)$

$\therefore -\theta = 90° + n(360°)$

$\therefore \theta = -90° + n(360°)$

$= 270° + n(360°)$ ◂

9 $\sin(x + 30°) = -\cos 2x$

$= -\sin(90° - 2x)$ *the reference angle*

$\therefore x + 30° = 180° + (90° - 2x) + n(360°),\ n \in \mathbb{Z}$ . . . **III**

$\therefore 3x = 240° + n(360°)$

$\therefore x = 80° + n(120°)$ ◂

OR : $x + 30° = 360° - (90° - 2x) + n(360°),\ n \in \mathbb{Z}$ . . . **IV**

$\therefore -x = 240° + n(360°)$

$\therefore x = -240° + n(360°)$

$\therefore x = 120° + n(360°)$ ◂

10 $\mathbf{2sin\ x\ cos\ x} - \sin x + 2\sin^2 x - \cos x = 0$

$2\sin x\cos x + 2\sin^2 x - \cos x - \sin x = 0$

$\therefore 2\sin x(\cos x + \sin x) - (\cos x + \sin x) = 0$

$\therefore (\cos x + \sin x)(2\sin x - 1) = 0$

$\therefore \cos x = -\sin x$ **OR** $\sin x = \frac{1}{2}$

$\div(-\cos x)$ $-1 = \tan x$ $\therefore x = 30° + n(360°),\ n \in \mathbb{Z}$ ◂

$\therefore x = 135° + n(180°),\ n \in \mathbb{Z}$ ◂ or $x = 150° + n(360°),\ n \in \mathbb{Z}$ ◂

11 $\mathbf{2sin\ x\ cos\ x} + \sin x - 6\cos x - 3 = 0$

$\therefore \sin x(2\cos x + 1) - 3(2\cos x + 1) = 0$

$\therefore (2\cos x + 1)(\sin x - 3) = 0$

$\therefore \cos x = -\frac{1}{2}$ . . . $\sin x \neq 3$!

$\therefore x = -120°$ ◂

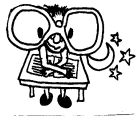
12 $\dfrac{\mathbf{2sin\ x\ cos\ x}}{\mathbf{1 - 2sin^2x - 1}} = 1$

$\therefore -\dfrac{\cos x}{\sin x} = 1$

INVERT: $\therefore -\tan x = 1$

$\therefore \tan x = -1$

$\therefore x = 135°$ ◂



13 $2.\mathbf{2sin\ x\ cos\ x} = 1$

$(\div 2)$ $\therefore \sin 2x = \frac{1}{2}$

$\therefore 2x = 30°$ or $150°$ . . . $\left\{ x \in [0°; 90°] \Rightarrow 2x \in [0°; 180°] \right\}$

$\therefore x = 15°$ or $75°$ ◂

14 $6\sin^2 x + 2.2\sin x\cos x = \mathbf{sin^2x + cos^2x}$

$\therefore 5\sin^2 x + 4\sin x\cos x - \cos^2 x = 0$

$\therefore (5\sin x - \cos x)(\sin x + \cos x) = 0$

$\therefore 5\sin x = \cos x$ or $\sin x = -\cos x$

$(\div \cos x)$ $\therefore 5\tan x = 1$ $\therefore \tan x = -1$

$\therefore \tan x = \frac{1}{5}$ (= 0,2) $\therefore x = -45°$ ◂

$\therefore x = 11{,}31°$ ◂